Journal of Engineering Physics and Thermophysics, Vol. 80, No. 1, 2007

HEAT CONDUCTION AND HEAT TRANSFER IN TECHNOLOGICAL PROCESSES

HEAT DISTRIBUTION IN A ROTATING RING–STATIONARY PIN TRIBOSYSTEM

A. Evtushenko[a,b] **and Yu. Tolstoi-Senkevich**[a]

UDC 539.377

We propose a computational scheme for calculating the temperature and the heat flow sharing coefficient at indentation of the end of a thin pin into the lateral surface of a rapidly rotating ring. It is supposed that on free lateral and end surfaces of the ring, as well as on the lateral surface of the pin, heat exchange by the Newton law occurs. The two-dimensional mixed quasi-stationary heat conduction problem for the ring has been solved by means of the finite integral Fourier transform. Formulas for calculating the temperature on the ring surface have been derived, and from the condition of equality of the average temperatures in the friction zone of the ring and the pin, the heat flow sharing coefficient has been determined. The influence of the ring thickness and its inner surface cooling on the maximum temperature has been investigated.

The two-dimensional quasi-stationary heat conduction problem for a homogeneous circular disk subjected to lateral surface heating by a fast-moving shared heat flow was solved by Gesim and Winer [1]. The pertinent heat conduction problem for a disk with a thin foreign coating applied to its lateral surface was solved in [2]. The case of two rotating heating zones on the surface of a continuous disk was considered in [3].

On the basis of these solutions the heat distribution between a rotating homogeneous [4, 5] and a piecewise homogeneous [6, 7] disk and a long pin indented into its lateral surface was investigated. The effect of the rotational velocity of the disk (Peclet parameter) and the heat transfer from the lateral surfaces (Biot criterion) of the disk and the pin on the heat flow sharing coefficient between the bodies and the maximum temperature in the contact region was investigated. In the present paper, the solution for a tribosystem consisting of a rotating circular ring and a thin long pin has been obtained.

Calculation Model. The rotating ring–stationary pin contact is schematically represented in Fig. 1. A circular cross-section pin is indented with one of its ends into the outer lateral surface of a ring by a constant force P which suffices to generate heat in the contact region of the bodies. It is assumed that:

1) the whole area of the pin end is in contact with a portion of the lateral surface of the ring;

2) the intensity of the heat flows into the ring and the pin is constant;

3) quasi-stationary heating conditions for the ring and stationary conditions for the pin take place;

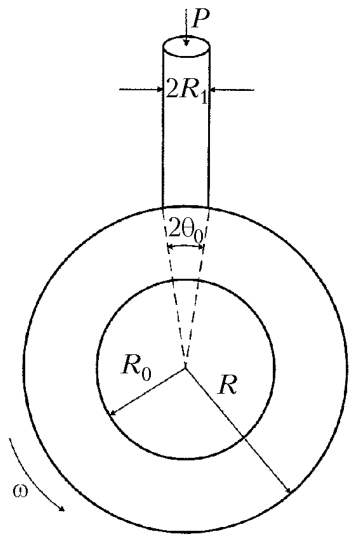
4) the ring temperature is a function of the radial and angular coordinates, the heat conduction of the ring in the axial direction is neglected;

5) high-speed friction heating conditions (Pe >> 100), under which the change in the temperature gradient in the angular direction can be neglected, is considered;

6) on the lateral and end surfaces of the ring, as well as on the lateral surface of the pin contact cooling occurs, the heat transfer coefficients are constant, and the respective Biot numbers are small;

7) the average temperature of the ring and the temperature of the pin in the contact region are equal.

[a]Bialystok Polytechnic Institute, Poland; [b]Ya. S. Podstrigach Institute of Applied Problems of Mechanics and Mathematics, NAS of Ukraine, 36 Nauchnaya Str., L'vov, 79061. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 80, No. 1, pp. 128–135, January–February, 2007. Original article submitted January 25, 2005; revision submitted June 8, 2005.



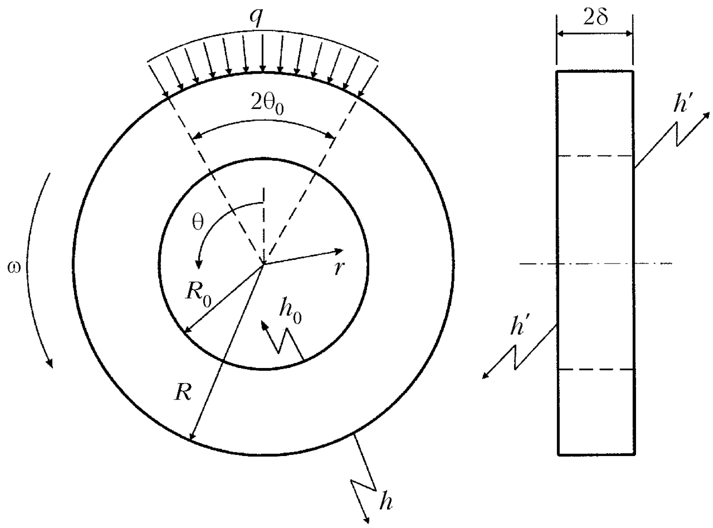


Fig. 1. Scheme of the rotating ring–stationary pin contact.

Fig. 2. Scheme of heating and cooling of the ring.

Formulation of the Heat Conduction Problem for the Ring. The heating and cooling scheme of the ring is shown in Fig. 2. In a polar coordinate system (r, θ) rigidly bound to the rotating heating region, on the basis of assumptions 1)–6), let us determine the temperature field $T(r, \theta)$ from the solution of the heat conduction equation [8]

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{h'}{K\delta} T = \frac{\omega}{k}\frac{\partial T}{\partial \theta} \tag{1}$$

under the following boundary conditions:

$$K \left.\frac{\partial T}{\partial r}\right|_{r=R} = \begin{cases} q, & |\theta| \le \theta_0; \\ -hT, & |\theta| > \theta_0, \end{cases} \tag{2}$$

$$K \left.\frac{\partial T}{\partial r}\right|_{r=R_0} = h_0 T. \tag{3}$$

Using dimensionless variables and parameters, let us write the boundary-value heat conduction problem (1)–(3) as

$$\frac{\partial^2 T^*}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial T^*}{\partial \rho} - \sigma T^* = \mathrm{Pe}\,\frac{\partial T^*}{\partial \theta}, \tag{4}$$

$$\left.\frac{\partial T^*}{\partial \rho}\right|_{\rho=1} = \begin{cases} 1, & |\theta| \le \theta_0; \\ -\mathrm{Bi}\,T^*, & |\theta| > \theta_0, \end{cases} \tag{5}$$

$$\left.\frac{\partial T^*}{\partial \rho}\right|_{\rho=\rho_0} = \mathrm{Bi}_0 T^*. \tag{6}$$

Solution of the Heat Conduction Problem for the Ring. Applying to the boundary-value heat conduction problem (4)–(6) the finite integral Fourier transform of the angular variable θ [9]

$$\overline{T}^* (\rho, n) = \frac{\varepsilon}{2\pi} \int_{-\pi}^{\pi} T^* (\rho, \theta) \exp(-in\theta)\, d\theta , \quad \varepsilon = \begin{cases} 1 , & n = 0 ; \\ 2 , & n \neq 0 , \end{cases}$$

taking into account the continuity conditions $T^*(\rho, -\pi) = T^*(\rho, \pi)$ and the smallness (in accordance with assumption 6)) of the product Bi T^*, we get

$$\frac{d^2 \overline{T}^* (\rho, n)}{d\rho^2} + \frac{1}{\rho} \frac{d\overline{T}^* (\rho, n)}{d\rho} - \lambda_n \overline{T}^* (\rho, n) = 0 , \tag{7}$$

$$\left. \frac{d\overline{T}^*}{d\rho} \right|_{\rho=1} = \frac{\varepsilon}{2\pi} \int_{-\theta_0}^{\theta_0} \exp(-in\theta)\, d\theta - \mathrm{Bi}\, \overline{T}^* (1, n) , \quad \left. \frac{d\overline{T}^*}{d\rho} \right|_{\rho=\rho_0} = \frac{\varepsilon}{2\pi} \mathrm{Bi}_0 \int_{-\pi}^{\pi} T^* (\rho_0, \theta) \exp(-in\theta)\, d\theta , \tag{8}$$

where $\lambda = \sqrt{\sigma + in\,\mathrm{Pe}}$.

The general solution of the differential equation (7) is of the form [10]

$$\overline{T}^* (\rho, n) = C(\lambda)\, I_0 (\lambda\rho) + D(\lambda)\, K_0 (\lambda\rho) , \tag{9}$$

where $C(\lambda)$ and $D(\lambda)$ are the unknown functions of the parameter λ. Substituting solution (9) into the transformed boundary conditions (8), we obtain

$$\overline{T}^* (\rho, n) = \frac{\varepsilon}{2\pi} \frac{L(\lambda, \rho)}{M(\lambda, \rho)} \int_{-\theta_0}^{\theta_0} \exp(-in\theta)\, d\theta , \tag{10}$$

$$L(\lambda, \rho) = I_0(\lambda\rho)\, [\lambda K_1 (\lambda\rho_0) + \mathrm{Bi}_0 K_0 (\lambda\rho_0)] + K_0(\lambda\rho)\, [\lambda I_1 (\lambda\rho_0) - \mathrm{Bi}_0 I_0 (\lambda\rho_0)] ,$$

$$M(\lambda, \rho) = [\lambda I_1 (\lambda\rho_0) + \mathrm{Bi}\, I_0 (\lambda)]\, [\lambda K_1 (\lambda\rho_0) + \mathrm{Bi}_0 K_0 (\lambda\rho_0)] +$$

$$+ [\lambda K_1 (\lambda\rho) - \mathrm{Bi}\, K_0 (\lambda)]\, [\lambda I_1 (\lambda\rho_0) - \mathrm{Bi}_0 I_0 (\lambda\rho_0)] .$$

Going over, with the aid of the rotation formula [9]

$$\overline{T}^* (\rho, \theta) = \mathrm{Re} \sum_{n=0}^{\infty} \overline{T}^* (\rho, n) \exp(in\theta)$$

in relations (10), to the originals, we determine the dimensionless temperature of the outer surface ($\rho = 1$) of the ring as

$$T^* (1, \theta) = T^*_{\mathrm{v}} + T^*_{\mathrm{f}} (1, \theta) . \tag{11}$$

The first term on the right-hand side of solution (11), which is independent of the angular coordinate, characterizes the volume temperature of the ring and is

$$T^*_{\mathrm{v}} = \frac{\theta_0 L_0}{\pi M_0} , \tag{12}$$

$$L_0 = I_0(\sqrt{\sigma})\,[\sqrt{\sigma}\,K_1(\rho_0\sqrt{\sigma}) + \mathrm{Bi}_0 K_0(\rho_0\sqrt{\sigma})] + K_0(\sqrt{\sigma})\,[\sqrt{\sigma}\,I_1(\rho_0\sqrt{\sigma}) - \mathrm{Bi}_0 I_0(\rho_0\sqrt{\sigma})]\,,$$

$$M_0 = [\sqrt{\sigma}\,I_1(\sqrt{\sigma}) + \mathrm{Bi}\,I_0(\sqrt{\sigma})]\,[\sqrt{\sigma}\,K_1(\rho_0\sqrt{\sigma}) + \mathrm{Bi}_0 K_0(\rho_0\sqrt{\sigma})] +$$

$$+ [\sqrt{\sigma}\,K_1(\sqrt{\sigma}) - \mathrm{Bi}\,K_0(\sqrt{\sigma})]\,[\sqrt{\sigma}\,I_1(\rho_0\sqrt{\sigma}) - \mathrm{Bi}_0 I_0(\rho_0\sqrt{\sigma})]\,.$$

Calculate the temperature burst T_f^* in (11) varying with angular coordinate by the formulas

$$T_\mathrm{f}^*(1, \theta) = \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{L_n(\theta)}{nM_n}\,, \tag{13}$$

where [11]

$$L_n(\theta) = (L_{n\mathrm{i}}M_{n\mathrm{r}} - L_{n\mathrm{r}}M_{n\mathrm{i}})\,C_n(\theta) - (L_{n\mathrm{r}}M_{n\mathrm{r}} + L_{n\mathrm{i}}M_{n\mathrm{i}})\,S_n(\theta)\,; \quad M_n = M_{n\mathrm{r}}^2 + M_{n\mathrm{i}}^2\,;$$

$$L_{n\mathrm{i}} = \exp(-c)\,[\sin(s)\,(d\cos(0.5\xi) - \mathrm{Bi}_0) - d\cos(s)\sin(0.5\xi)] - d\sin(0.5\xi)\,;$$

$$L_{n\mathrm{r}} = \exp(-c)\,[\cos(s)\,(d\cos(0.5\xi) - \mathrm{Bi}_0) + d\sin(s)\sin(0.5\xi)] + d\cos(0.5\xi) + \mathrm{Bi}_0\,;$$

$$M_{n\mathrm{r}} = \exp(-c)\,\Big\{\cos(s)\,[\sigma - (\mathrm{Bi} - \mathrm{Bi}_0)\,d\cos(0.5\xi) - \mathrm{Bi}\mathrm{Bi}_0] -$$

$$- \sin(s)\,[(\mathrm{Bi} - \mathrm{Bi}_0)\,d\sin(0.5\xi) - n\,\mathrm{Pe}]\Big\} - \sigma - (\mathrm{Bi} - \mathrm{Bi}_0)\,d\cos(0.5\xi) - \mathrm{Bi}\mathrm{Bi}_0\,;$$

$$M_{n\mathrm{i}} = \exp(-c)\,\Big\{\cos(s)\,[(\mathrm{Bi} - \mathrm{Bi}_0)\,d\sin(0.5\xi) - n\mathrm{Pe}] +$$

$$+ \sin(s)\,[\sigma - (\mathrm{Bi} - \mathrm{Bi}_0)\,d\cos(0.5\xi) - \mathrm{Bi}\mathrm{Bi}_0]\Big\} + n\mathrm{Pe} + (\mathrm{Bi} + \mathrm{Bi}_0)\,d\sin(0.5\xi)\,;$$

$$C_n(\theta) = \cos[n(\theta_0 + \theta)] - \cos[n(\theta_0 - \theta)]\,; \quad S_n(\theta) = \sin[n(\theta_0 + \theta)] + \sin[n(\theta_0 - \theta)]\,;$$

$$d = (\sigma^2 + n^2\mathrm{Pe}^2)^{0.25}\,; \quad s = 2d\,(1 - \rho_0)\sin(0.5\xi)\,; \quad c = 2d\,(1 - \rho_0)\cos(0.5\xi)\,; \quad \xi = \arctan(n\mathrm{Pe}/\sigma)\,.$$

Friction Flow Sharing Coefficient. According to assumption 7), in the contact region of the rotating ring and the stationary pin the equality

$$T_\mathrm{a} = T_\mathrm{p}\,, \tag{14}$$

where

$$T_\mathrm{a} = \frac{1}{2\theta_0}\int_{-\theta_0}^{\theta_0} T(1, \theta)\,d\theta \tag{15}$$

is fulfilled.
Substituting the expressions for the temperature of the outer surface of the ring (11)–(13) into relation (15), upon integration we obtain

$$T_\mathrm{a} = \frac{qR}{K}\,T_\mathrm{a}^*\,, \tag{16}$$

where

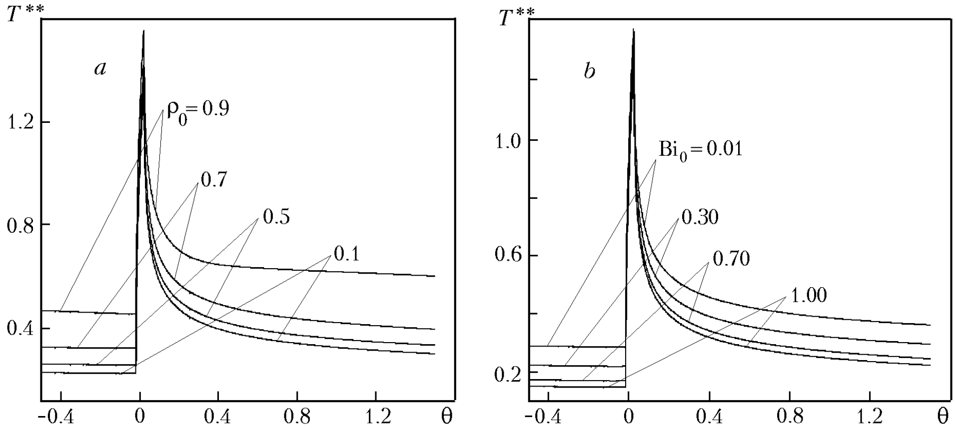


Fig. 3. Dimensionless temperature T^{**} distribution over the ring surface: a) for various values of the dimensionless inner radius ρ_0 and at a fixed Biot criterion Bi_0 = 0.1; b) for various values of the Biot criterion Bi_0 and at a constant dimensionless thickness of the ring ρ_0 = 0.5.

$$T_a^* = \frac{1}{\pi}\left[\frac{\theta_0 L_0}{M_0} - \frac{2}{\theta_0}\sum_{n=1}^{\infty}\frac{(L_{nr}M_{nr} + L_{ni}M_{ni})\sin^2(n\theta_0)}{n^2 M_n}\right]. \tag{17}$$

The solution of the one-dimensional stationary heat conduction equation for a round long pin subjected to heating at one end by a flow of intensity q_p and to convective cooling on the lateral surface is given in [5]. The temperature of the pin end rubbing against the ring surface is

$$T_p = \frac{q_p l}{K_p \varphi}\tanh\varphi . \tag{18}$$

Substituting temperatures (16)–(18) into equality (14), we get the heat flow sharing coefficient

$$\eta = \frac{\gamma}{1+\gamma}, \quad \gamma \equiv \frac{Q}{Q_p} = \frac{2\sqrt{2}\,K\Delta\tanh(\varphi)}{\pi K_p\sqrt{Bi_p}\,T_a^*}, \quad 0 \le \eta \le 1 , \tag{19}$$

and the case of η = 0 thereby corresponds to such heat generation conditions under which the friction heat is expended in heating the pin, and at η = 1 — in heating the ring.

Since the total quantity of heat generated in the friction zone is [12]

$$Q + Q_p = f\omega RP ,$$

in view of equality (19) the formula for determining the surface temperature of the ring has the form

$$T(R, \theta) = \eta\,\frac{f\omega RP}{4\delta\theta_0 K}\,T^*(1, \theta) ,$$

where T^* is the dimensionless temperature of the outer surface of the ring (11)–(13).

Numerical Results. In the limiting case, $\rho_0 \to 0$, $Bi_0 \to 0$, and from formulas (11)–(13) the solution of the respective heat conduction equation for a continuous disk follows [4, 5]. Therefore, let us investigate the dependence of the temperature and heat distribution coefficient on two new dimensionless parameters: internal radius of the ring ρ_0 and Biot criterion Bi_0. In the calculations, we recorded the dimensionless parameters Bi = Bi′ = 0.1; K/K_p = 1; l/R_p = 6; Δ = 0.1. The data presented in Figs. 3–5 have been obtained for the angular half-length of the contact area θ_0 = 0.02, and in Figs. 3, 4, 6 — for Pe = 100.

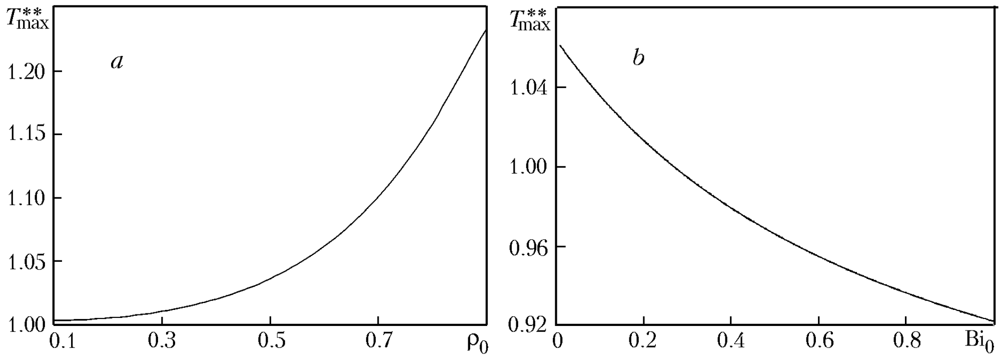


Fig. 4. Maximum dimensionless temperature T^{**}_{max} versus the dimensionless radius ρ_0 at a fixed Biot criterion $Bi_0 = 0.1$ (a) and the Biot criterion Bi_0 at a constant dimensionless thickness of the ring $\rho_0 = 0.5$ (b).

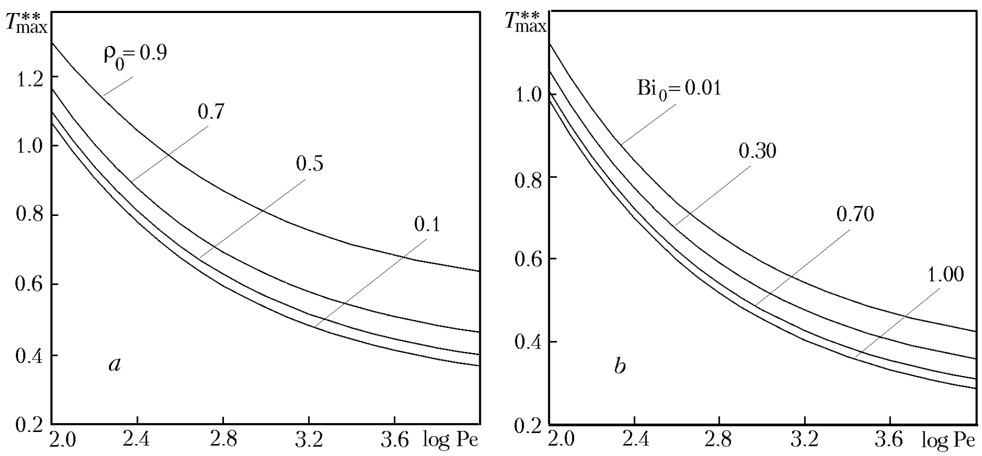


Fig. 5. Maximum dimensionless temperature T^{**}_{max} versus the Peclet parameters at various values of: a) dimensionless inner radius of the ring ρ_0 at a fixed Biot criterion $Bi_0 = 0.1$; b) the Biot criterion at a constant value of the dimensionless thickness of the ring $\rho_0 = 0.5$.

The dimensionless temperature distribution $T^{**} = T^{*}/\theta_0$ over the working (outer) surface of the ring for various values of the parameter ρ_0 is given in Fig. 3a, and for several values of the Biot criterion Bi_0 — in Fig. 3b. It is seen that before the contact region the temperature smoothly increases from the level of the volume temperature of the ring (12). As the friction zone is approached, the temperature sharply increases, reaching its maximum value at the entrance point $\theta = \theta_0$. Then there is a swift temperature drop to the level of the volume temperature. With increasing inner radius of the ring (decreasing thickness) the temperature increases (Fig. 3a). The influence of convective cooling of the inner surface of the ring at its fixed thickness on the working surface temperature is shown in Fig. 3b. With increasing Biot criterion Bi_0 both the volume temperature and the burst temperature decrease.

These effects are given more clearly in Fig. 4, which shows the dependence of the maximum dimensionless temperature $T^{**}_{max} = T^{*}(1, \theta_0)$ on the parameter ρ_0 (Fig. 4a) and the Biot parameter Bi_0 (Fig. 4b). Note that for values $0 \le \rho_0 \le 0.15$ the temperature practically agrees with the respective results for the continuous disk [4, 5].

The influence of the rotational velocity of the ring (Peclet parameter Pe) on the maximum temperature in the friction zone is shown in Fig. 5. The data presented in Fig. 5a have been obtained for various thickness values of the ring at a constant cooling intensity of its inner surface, and in Fig. 5b — vice versa, at a constant thickness of the ring and various values of the Biot criterion Bi_0. The common thing for both these figures is the fact that with increasing Peclet parameter the maximum dimensionless temperature of the ring surface decreases. Earlier this effect was noted for a disk in [4]–[7].

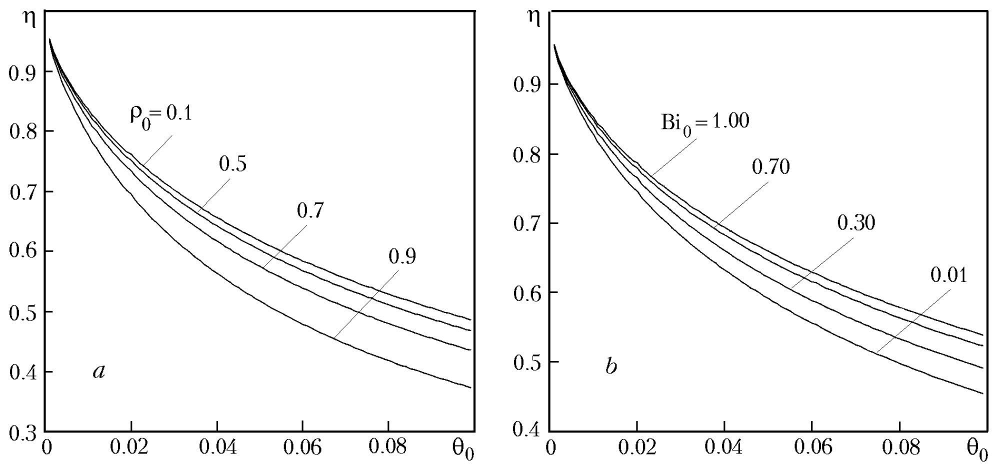


Fig. 6. Heat flow sharing coefficient η versus the angular half-length of the contact region θ_0 for various values of: a) the dimensionless inner radius of the ring ρ_0 at a fixed Biot criterion $\mathrm{Bi}_0 = 0.1$; b) the Biot criterion at a constant value of the dimensionless thickness of the ring $\rho_0 = 0.5$.

The dependences of the heat distribution coefficient η (19) on the angular half-length of the friction zone θ_0 are given in Fig. 6. It is seen that with decreasing dimensions of the contact region the portion of heat expended in heating the pin increases. With expanding friction area the total quantity of heat directed into the ring increases.

Thus, the investigation of the influence of the ring thickness and cooling of its inner surface on the maximum temperature in the friction zone and the friction heat distribution between the ring and the pin has shown that:

a) for values of the inner radius of the ring R_0 not exceeding 0.15 of its outer radius R, to calculate the temperature conditions of the ring–pin pair, one can use the known, simpler solution for the disk–pin system [4];

b) the highest temperature in the friction zone is attained in the case of heat insulation of the inner surface of the ring (at its fixed thickness); with enhancing heat exchange on the above surface the ring temperature decreases;

c) an increase in the angular rotational velocity of the disk leads to a decrease in the maximum temperature, and for a given rotational velocity the temperature increases with decreasing thickness of the ring;

d) an increase in the contact region sizes leads to an increase in the portion of heat entering the ring; for a given angular width of the friction zone, a decrease in the ring thickness leads to an increase in the quantity of heat needed for heating the pin, and at a fixed thickness an increase in the inner surface cooling intensity causes an increase in the quantity of heat entering the ring.

This work is part of the scientific-research project W/WM/1/04 executed at the Bialystok Polytechnic Institute, Poland.

NOTATION

$A \cong 4\delta R\theta_0$, area of the contact region, m^2; $A_p = \pi R_p^2$, cross-section area of the pin, m^2; $\mathrm{Bi} = hR/K$, Biot criterion for the outer surface of the ring; $\mathrm{Bi}_0 = h_0 R/K$, Biot criterion for the inner surface of the ring; $\mathrm{Bi}' = h'R/K$, Biot criterion for the end surfaces of the ring; $\mathrm{Bi}_p = h_p R_p/K_p$, Biot criterion for the lateral surface of the pin; h, heat transfer coefficient on the outer surface of the ring, W/(m^2·K); h_0, heat transfer coefficient on the inner surface of the ring, W/(m^2·K); h', heat transfer coefficient on the end surfaces of the ring, W/(m^2·K); h_p, heat transfer coefficient from the lateral surface of the pin, W/(m^2·K); $I_k(\cdot)$, modified Bessel function of the first kind of order k; k, thermal diffusivity of the ring, m^2/sec; K, heat conductivity coefficient of the ring, W/(m·K); K_p, heat conductivity coefficient of the pin, W/(m·K); $K_k(\cdot)$, modified Bessel function of the second kind of order k; l, length of the pin, m; n, nonnegative integer; P, pressing force, N; $\mathrm{Pe} = \omega R^2/k$, Peclet parameter; q, intensity of the heat flow into the ring, W/m^2; q_p, intensity of the heat flow into the pin, W/m^2; Q, qA, quantity of heat entering the ring, W; $Q_p = q_p A_p$, quantity of heat entering the pin, W; r, radial coordinate, m; R, outer radius of the ring, m; R_0, inner radius of the ring, m; R_p, radius of the pin cross-section, m; T, temperature, K; T_a, average temperature of the ring in the friction zone, K;

T_p, temperature on the working end of the pin, K; $T^* = kT/(qR)$, dimensionless temperature of the ring; $T^{**} = T^*/\theta_0$, dimensionless temperature; δ, ring thickness, m; $\Delta = \delta/R$, dimensionless thickness of the ring; $\varphi = \sqrt{2\,\mathrm{Bi}_p}\,l/R_p$, dimensionless parameter; η, heat flow sharing coefficient; θ, angular coordinate, rad; θ_0, angular half-width of the contact region, rad; $\rho = r/R$, dimensionless radial coordinate; $\rho_0 = R_0/R$, dimensionless inner radius of the ring; $\sigma = \mathrm{Bi}'/\Delta$, dimensionless parameter; ω, rotational velocity of the ring, rad/sec. Subscripts: a, averaged temperature; f, flash point; *k*, order of Bessel functions; max, maximum values; *n*, summation in Fourier series; p, pin; r and i, real and imaginary parts of a function; v, volume temperature; *, dimensionless quantities; 0, parameters pertaining to the inner surface of the ring and angular length of the friction zone; overscribed bar, transformed quantity.

REFERENCES

1. B. Gecim and W. O. Winer, Steady temperature in a rotating cylinder subject to surface heating and convective cooling, *Trans. ASME J. Tribol.*, **106**, No. 1, 120–127 (1984).
2. B. Gecim and W. O. Winer, Effect of a surface film on the surface temperature of a rotating cylinder, *Trans. ASME J. Tribol.*, **108**, No. 1, 92–97 (1986).
3. B. Gecim and W. O. Winer, Steady temperature in a rotating cylinder — some variations in the geometry and the thermal boundary conditions, *Trans. ASME J. Tribol.,* **108**, No. 2, 446–454 (1986).
4. A. Yevtushenko, O. Ukhanska, and R. Chapowska, Friction heat distribution between a stationary pin and rotating disc, *Wear*, **196**, Nos. 1, 2, 219–225 (1996).
5. A. A. Yevtushenko, O. M. Ukhanskaya, and R. B. Chapowska, Friction-heat distribution in a pin–disk tribosystem, *Inzh.-Fiz. Zh.*, **70**, No. 3, 500–505 (1997).
6. A. Yevtushenko and R. Chapowska, Effect of a thin film on the heat distribution between a stationary pin and a rotating disk, *Num. Heat Transfer*, Pt. A, **30**, No. 8, 839–848 (1996).
7. A. A. Yevtushenko and R. B. Chapowska, Influence of a coating on the heat distribution in the pin–disk tribosystem, *Fiz. Khim. Mekh. Mater.*, **33**, No. 3, 43–49 (1997).
8. E. M. Kartashov, *Analytical Methods in the Theory of the Heat Conduction of Solids* [in Russian], Nauka, Moscow (1985).
9. I. N. Sneddon, *The Use of Integral Transforms*, Academic Press, New York (1972).
10. B. G. Korenev, *The Problems of Thermal Conductivity and Thermal Elasticity* [in Russian], Nauka, Moscow (1980).
11. M. Abramovitz and I. A. Stegun (Eds.), *Handbook of Mathematical Functions with Formulas*, *Graphs, and Mathematical Tables* [Russian translation], Nauka, Moscow (1979).
12. K. L. Johnson, *Contact Mechanics* [Russian translation], Mir, Moscow (1989).